Weinberg's[5] treatment of light deflection, equal contributions are found from the $A(r)$ and $B(r)$ terms in the Schwartzschild metric:

$$d\tau^2 = B(r)dt^2 - A(r)dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \quad (14)$$

where

$$B(r) = 1 - \frac{2MG}{r} + \cdots, \quad (15)$$

$$A(r) = 1 + \frac{2MG}{r} + \cdots. \quad (16)$$

The effects of the equivalence principle and special relativity are contained only in $B(r)$. In fact, the gravitational redshift originates entirely from this term.[6] The purely general relativistic effect of space curvature originates from $A(r)$. This effect is evidently not taken into account in the perturbative scheme proposed in Ref. 1.

ACKNOWLEDGMENT

I thank A. Alaniz for useful discussions of his paper.

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Where is the "Wien peak"?

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(Received 30 May 2003; accepted 3 July 2003)

[DOI: 10.1119/1.1604387]

The analysis and interpretation of continuous spectra has a reputation for leading people astray.[1] The intensity function for blackbody radiation can be presented in *wavelength* bookkeeping ("intensity per unit wavelength-increment"), or in *frequency* bookkeeping ("intensity per unit frequency-increment"). As is well known, the maxima of these curves occur at different wavelengths (or frequencies, $\nu = c/\lambda$) even though they represent the same physical spectrum. The fact that the wavelength bookkeeping places the "peak" of the Sun's spectrum at about 500 nm, remarkably close to the maximum sensitivity of the human eye, leads to perennial speculation of a causal link in human evolution. Although this hypothesis has been well debunked,[2] a recent paper raises the question once again.[3] (The Sun's "peak" by the frequency bookkeeping comes at about 880 nm.)

To present a continuous spectrum such as blackbody radiation—either graphically or as a formula—requires an essentially arbitrary choice of the *dispersion rule*, most commonly the familiar linear-wavelength and linear-frequency bookkeepings cited above [with integration differentials $d\lambda$ and $d\nu = (-)(c/\lambda^2)d\lambda$, respectively]. But nature provides no physical (or biological) preference between these two, and indeed there are other useful dispersion rules. For instance, one can work with "intensity per percentage bandwidth" (i.e., the logarithm of either wavelength or frequency). For this *logarithmic* dispersion rule, the integration differential becomes

$$(-)d(\ln\lambda) = d(\ln\nu) = (1/\nu)d\nu = (-)(1/\lambda)d\lambda. \quad (1)$$

The traditional physicists' preference for the wavelength dispersion rule comes from the fact that spectrometers using *diffraction gratings* give experimental dispersion that approximates the linear-wavelength rule. It can be shown that the other common class of spectrometers, those using *prisms*, give an experimental dispersion that approximates a *frequency-squared* rule; that is, the spectrum is spread out as "intensity per increment in frequency-squared," and the integration differential is $d(\nu^2) = 2\nu\, d\nu$.

All four of these dispersion rules, with their corresponding graphs and formulas, represent the same physical spectrum. Traditionally the name "Wien's displacement law" is applied specifically to the peak reckoned by the wavelength-dispersion bookkeeping:

$$\lambda_{\max}T = hc/(4.97...)k = 2.90 \text{ mm K} \quad \text{(wavelength rule [}\sim\text{grating]).} \quad (2)$$

By the other dispersion bookkeepings, the analogous "Wien peaks" occur at:

$$\lambda_{\max}T = hc/(3.92...)k = 3.67 \text{ mm K} \quad \text{(logarithmic rule)} \quad (3)$$

$$= hc/(2.82...)k = 5.10 \text{ mm K} \quad \text{(linear frequency rule)} \quad (4)$$

$$= hc/(1.59...)k = 9.03 \text{ mm K} \quad \text{(frequency-squared rule [}\sim\text{prism]).} \quad (5)$$

The numbers in the denominators of these expressions are the roots of the equation $\exp(-x) = 1 - (x/m)$, where $x = hc/kT\lambda_{\max}$, for $m = 5,4,3,2$, respectively [cf. Eqs. (2)–(5) of Ref. 3]. When the blackbody spectrum recorded by a grating spectrometer is compared with the same spectrum recorded by a prism spectrometer, the apparent "peak wavelengths" differ (approximately) by the ratio of 4.97/1.59

$=3.12$—a factor that is significantly larger than the human visual bandwidth!

There is yet another way to define the ''Wien peak''—one that *is* independent of the artifact of the dispersion rule chosen to display the spectrum. This is to integrate the Planck function (in any consistent bookkeeping) to find the frequency or wavelength such that *one-half* of the total radiation intensity falls on either side.[4] The result leads to

$$\lambda_{max}T = hc/(3.50302...)k = 4.11 \ \ \mathrm{mm\,K} \qquad (\text{``50\%'' or ``median'' rule}). \quad (6)$$

This definition is arguably the most physically meaningful numerical criterion for the ''Wien peak'' of the blackbody spectrum, and should be more widely taught. It places the Sun's peak at about 710 nm, at the extreme red end of human vision.

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