

# Depinning of Domain Walls in a Notched Ferromagnetic Nanostrip: Role of Inertial and Nonlinear Damping Effects

Sarabindu Dolui<sup>1</sup>, Sumit Maity<sup>1</sup>, Sharad Dwivedi<sup>1</sup>, and Giancarlo Consolo<sup>2</sup>

<sup>1</sup>Department of Mathematics, School of Sciences, National Institute of Technology Andhra Pradesh, Tadepalligudem, 534 101, India

<sup>2</sup>Department of Mathematics, Computer, Physical and Earth Sciences, University of Messina, 98166 Messina, Italy

In this work, we investigate theoretically the static and kinetic depinning field of a domain wall in a notched magnetic nanostrip under the generalized framework of the Landau–Lifshitz–Gilbert equation, which combines inertial and nonlinear viscous-dry friction damping effects. We assume a head-to-head transverse domain wall configuration and analyzed its motion subject to an external magnetic field. To deduce the equation ruling the spatio-temporal evolution of the magnetic domain wall, we adopt the Schryer and Walker trial function approach. The results show that static and kinetic depinning fields increase as the dry friction dissipation increases. Moreover, viscous dissipation exhibits a weak dependence on the kinetic depinning field and saturated domain wall velocity, while inertial damping due to the relaxation time of angular momentum significantly impacts the kinetic depinning field, depinning time, and breakdown velocity. Our numerical results are in good qualitative agreement with the recent observations reported in the literature.

*Index Terms*—Domain wall, Static depinning, Kinetic depinning, Viscous dissipation, Dry-friction dissipation, Inertial damping.

## I. INTRODUCTION

OVER the recent years, the ability to control the domain wall (DW) motion in ferromagnetic nanostructures has sparked a wide range of applications such as the next generation of memory state, magnetic sensors, logic devices, racetrack memories, etc. As these sophisticated applications continue to advance, there has been a need for DW-based, cost-effective electronic storage devices with less space consumption. As a result, ferromagnetic nanostructures (nanostrips/nanotubes) have emerged as a viable option for recording and storing data in these modern spintronic devices. In order to improve the performance of these devices and achieve higher processing speeds, it is necessary to gain a deep understanding of DW (the transition zone that separates the uniform magnetization regions, referred to as domains) motion in the presence of external sources. Specifically, the precise control over DW position.

In particular, a notch functions as a local pinning center for the magnetic walls, pivotal in controlling and modulating their behavior. The effective pinning field produced by the notch acts as an energetically significant potential well (or barrier) and represents an additive contribution to the overall effective field experienced by magnetic DWs. The specific zone around the notch wherein the DW is pinned or temporarily trapped is called the pinning field region. Motivated by the studies [1,2,3,4], the present work explores the combined action of an applied magnetic field, nonlinear dry friction-viscous dissipation, and inertial damping on DW depinning in a notched nanostructure. The main focus is to analyze two distinct scenarios of DW depinning: (i) static depinning, where the DW is initially pinned at the notch, and (ii) kinetic depinning, where a moving DW encounters the notch.

## II. MICROMAGNETIC MODEL

We consider a notched ferromagnetic nanostrip of length  $L_x$ , width  $L_y$ , and thickness  $L_z$  along the  $\mathbf{e}_1$ ,  $\mathbf{e}_2$  and  $\mathbf{e}_3$  axes, respectively. As depicted in Fig. 1, a head-to-head TDW of width  $\Delta$  is nucleated along the strip-axis ( $\mathbf{e}_1$ -direction), separating two faraway domains that point along  $\pm\mathbf{e}_1$  directions.

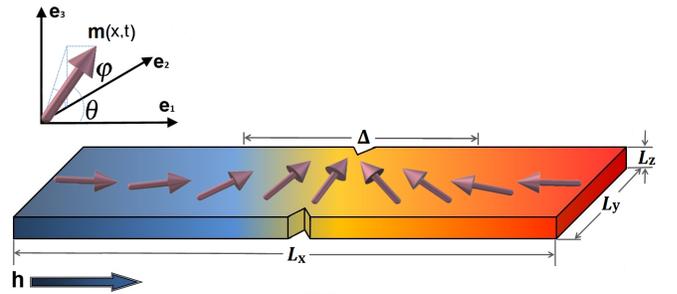


Fig. 1. Schematics of a ferromagnetic nanostrip with an artificial notch in the form of two symmetrical triangles and reference axes..

### A. Governing Equation

The spatiotemporal evolution of the local magnetization vector  $\mathbf{m}(x, t)$  within the ferromagnetic medium is governed by the inertial Landau-Lifshitz-Gilbert (iLLG) equation, which can be expressed as:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma (\mathbf{m} \times \mathbf{h}_{\text{eff}}) + \mathbf{T}_{\text{damp}} + \mathbf{T}_{\text{inertia}}, \quad (1)$$

where  $\mathbf{m}(x, t) = \mathbf{M}(x, t)/M_s$  describes the normalized magnetization vector field  $\mathbf{m} : \mathbb{R} \times \mathbb{R}^+ \rightarrow \mathbb{S}^2$ , where  $\mathbf{M}$  is magnetization vector field,  $M_s$  represents the saturation magnetization, ‘ $\times$ ’ denotes the vector product on  $\mathbb{R}^3$ , and  $\mathbb{S}^2$  stands for a unit sphere in  $\mathbb{R}^3$ .

The total effective field,  $\mathbf{h}_{\text{eff}}$  comprises contributions from the exchange  $\mathbf{h}_{\text{ex}}$ , anisotropy  $\mathbf{h}_{\text{ani}}$ , external  $\mathbf{h}_{\text{ext}}$ , demagnetization (stray)  $\mathbf{h}_{\text{dmg}}$  and the pinning  $\mathbf{h}_{\text{pin}}$ .

The local pinning field  $\mathbf{h}_{\text{pin}} = h_p(x)\mathbf{e}_1$  is induced by the notch located at  $x = 0$ , leading to a localized enhancement of the easy-axis anisotropy. Here,  $h_p(x)$  defines the spatial variation of pinning field, which is given by:

$$h_p(x) = -\frac{1}{2\mu_0 M_s^2 L_y L_z} \frac{\partial V_{\text{pin}}(x)}{\partial x}, \quad (2)$$

where,  $V_{\text{pin}}(x)$  is the space-dependent pinning potential of magnetostatic nature, which can be described by:

$$V_{\text{pin}}(x) = \begin{cases} \frac{1}{2} K_N x^2, & \text{for } |x| \leq L_N \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Here,  $K_N$  represents the elastic constant of the constriction.

The damping torque and inertial torque are defined as follows:

$$\mathbf{T}_{\text{damp}} = \left[ \alpha_G \left( 1 + \frac{\alpha_v}{\gamma^2} \left( \frac{\partial \mathbf{m}}{\partial t} \right)^2 \right) + \gamma \alpha_d \left| \frac{\partial \mathbf{m}}{\partial t} \right|^{-1} \right] \left( \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \right), \quad (4)$$

$$\mathbf{T}_{\text{inertia}} = \alpha_G \tau \left[ 1 + \frac{\alpha_v}{\gamma^2} \left( \frac{\partial \mathbf{m}}{\partial t} \right)^2 \right] \left( \mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial t^2} \right), \quad (5)$$

where  $\alpha_G$  and  $\alpha_v$  are the linear Gilbert and nonlinear viscous damping coefficients, respectively. The parameter  $\alpha_d$  is a positive phenomenological dry friction coefficient that captures the average effect of crystallographic defects in the material. The parameter  $\tau$  denotes the relaxation time of angular momentum.

### III. NUMERICAL RESULTS

In order to investigate the pinning and depinning of DWs, we consider a ferromagnetic nanostrip characterized by the material parameters of a cobalt-platinum-chromium (CoPtCr) alloy.

### IV. CONCLUSION

Our results indicate that nonlinear viscous-dry friction and inertial damping play a vital role and yield an additional degree of freedom to control the DW position and its velocity precisely in a ferromagnetic nanostrip via geometrical notches. More precisely, ferromagnetic materials with low dry friction dissipation coefficients are more advantageous for developing energy-efficient high-speed devices. Moreover, our findings suggest that maintaining a lower relaxation time of angular momentum is favorable for achieving a low kinetic depinning field, which can enhance the performance of such devices.

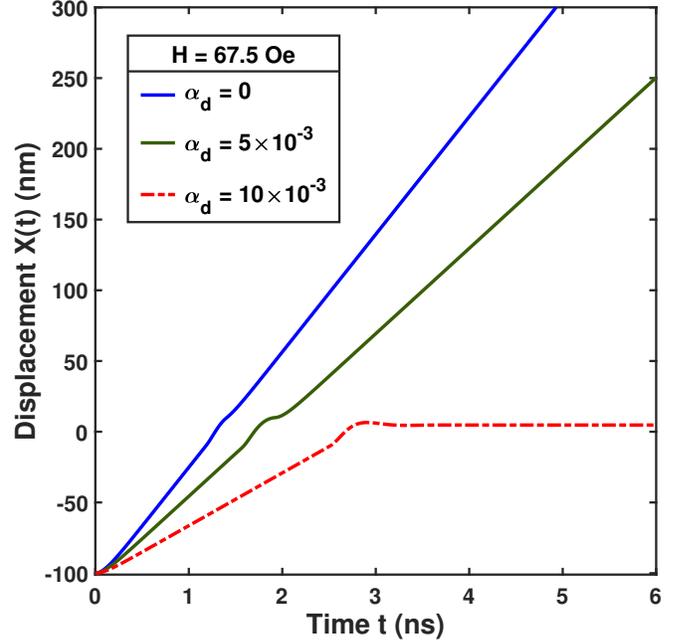


Fig. 2. The dependence of DW displacement  $X(t)$  on time  $t$ , varying  $\alpha_d$ .

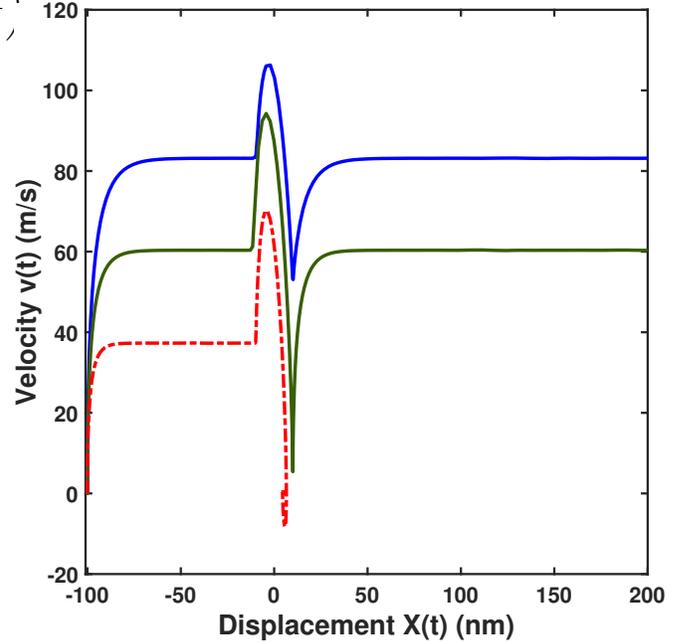


Fig. 3. The dependence of DW velocity  $v(t)$  on DW displacement  $X(t)$ , varying  $\alpha_d$ .

### REFERENCES

- [1] S. S. Parkin, M. Hayashi, and L. Thomas, "Magnetic domain-wall racetrack memory," *science*, vol. 320, no. 5873, pp. 190–194, 2008.
- [2] M. Kläui, H. Ehrke, U. Rüdiger, T. Kasama, R. E. Dunin-Borkowski, D. Backes, L. J. Heyderman, C. A. Vaz, J. A. C. Bland, G. Faini *et al.*, "Direct observation of domain-wall pinning at nanoscale constrictions," *Applied Physics Letters*, vol. 87, no. 10, 2005.
- [3] K. Neeraj, M. Pancaldi, V. Scalera, S. Perna, M. d'Aquino, C. Serpico, and S. Bonetti, *Magnetization switching in the inertial regime*. APS, 2022, vol. 105, no. 5.
- [4] S.-M. Ahn, K.-W. Moon, D.-H. Kim, and S.-B. Choe, "Detection of the static and kinetic pinning of domain walls in ferromagnetic nanowires," *Applied Physics Letters*, vol. 95, no. 15, 2009.